Algebra 2 Mid-Year Test
Study Guide

Name: KEY

Date: _____ Block: _____

$i \to I$ $\quad i^3 = -i$ $\quad$ "I Won I Won!"

$i^2 = -1$ $\quad i^4 = 1$

Topic 3: Complex Numbers

***Recall:** $i = \sqrt{-1}$ $\quad i^2 = -1$ $\quad i^3 = -i$ $\quad i^4 = 1$

Simplify each expression.

18. $\sqrt{-81}$

$= \sqrt{-1}\cdot\sqrt{81}$

$= i9 = 9i$

19. $2\sqrt{-14}\cdot\sqrt{8}$

$7\ 2\ 2\ 4$

$= 2(4)i\sqrt{7}$

$= 8i\sqrt{7}$

20. $18i^{49}$

$= 18(i^4)^{12}\cdot i$

$= 18i$

21. $(i^3\sqrt{5})^2\cdot 2i$

$i^6(5)(2)i = 10i^7$

$-10i$

22. $(4-7i)-(5+2i)$

$= 4-7i-5-2i$

$= \boxed{-1-9i}$

23. $(9+2i)(4-i)$

$36-9i+8i-2i^2$

$36-i+2 = \boxed{38-i}$

24. $(6-2i)^2$

$= (6-2i)(6-2i) =$

$36-12i-12i+4i^2 =$

$36-24i-4 = \boxed{32-24i}$

25. $\frac{15}{6i}\cdot\frac{i}{i} = \frac{15i}{6i^2}$

$= \frac{15i}{-6} = \boxed{\frac{5i}{-2}}$

26. $\frac{(1-5i)}{(2i)}\frac{i}{i} = \frac{i-5i^2}{2i^2}$

$= \frac{i+5}{-2} = \boxed{\frac{5+i}{-2}}$

27. $\frac{(3-3i)}{(7-i)}\frac{(7+i)}{(7+i)} =$

$= \frac{21+3i-21i-3i^2}{49-i^2}$

$= \frac{21-18i+3}{49+1} = \frac{24-18i}{50}$

or $\frac{12-9i}{25}$

Topic 4: Classifying Numbers & Identifying Properties

Simplify (if possible), ~~then name all sets to which each value belongs.~~

28. $\frac{\sqrt{64}}{5} = \frac{8}{5}$

29. $\sqrt{-36}$

$= 6i$

30. $\sqrt{3}-\sqrt{3} = 0$

31. $|1-4^2|$

$= |1-16| = |-15|$

$= 15$

32. $2+5i$

33. $\sqrt{15}$

3.9

34. $-\sqrt{\frac{45}{5}} = -\sqrt{9}$

$= \boxed{-3}$

35. $-8i\cdot 5i$

$= -40i^2$

$= -40(-1) = 40$

Name the property that justifies each statement.

36. $\sqrt{5}(8-\sqrt{3}) = 8\sqrt{5} - \sqrt{3}\cdot\sqrt{5}$

Distributive

37. $2m^3 + (-2m^3) = 0$

Additive Inverse.

38. $-9\cdot(2+3i) = (2+3i)\cdot -9$

Commutative

39. $\left(\frac{x}{5}\right)\cdot\left(\frac{5}{x}\right) = 1$

Multiplicative Inverse

Topic 5: Factoring

Identify the special factoring patterns.

DIFFERENCE OF SQUARES	~~SUM OF CUBES~~	~~DIFFERENCE OF CUBES~~
$a^2 - b^2 =$	$a^3 + b^3 =$	$a^3 - b^3 =$
$(a+b)(a-b)$		

Factor the expressions below completely.

40. $36x^2 - 64y^2$

$(6x)^2 - (8y)^2 =$

$\boxed{(6x+8y)(6x-8y)}$

41. ~~$m^3 + 64$~~ $2m^2 - 200$

$= 2(m^2 - 100)$

$= \boxed{2(m+10)(m-10)}$

42. $81k^4 - 3k^2$

$3k^2(27k^2 - 1)$

43. $2xy^5 + 250xy^3$

$2xy^3(y^2 + 125)$

44. $p^3 - 10p^2 + 25p$

$= p(p^2 - 10p + 25) =$

$= p(p-5)(p-5) =$

$= \boxed{p(p-5)^2}$

$-5 \quad ? \quad 25 \quad ? \quad -5$, -10

45. $x^4 - 11x^2 + 28$

$(x^2-7)(x^2-4)$

$\boxed{(x^2-7)(x+2)(x-2)}$

$-7 \quad ? \quad 28 \quad ? \quad -4$, -11

46. $4x^3 + 18x^2 - 10x$

$2x(2x^2 + 9x - 5)$

$\boxed{2x(2x-1)(x+5)}$

$-x$

$+10x$

$(9x)$ ✓

47. $9x^3 - 63x^2 - x + 7$

$= 9x^2(x-7) - 1(x-7)$

$= (x-7)(9x^2-1)$

$= \boxed{(x-7)(3x-1)(3x+1)}$

Discriminant Formula:

- If $d < 0$, there are **2 imaginary roots.**
- If $d = 0$, there is **1 rational root**
- If $d > 0$ **and a perfect square**, there are **2 rational roots**
- If $d > 0$ **and not a perfect square**, there are **2 irrational roots.**

Find the discriminant of each equation. Then, determine the number and type of roots.

54. $x^2 - 7x + 15 = 0$

discriminant $= b^2 - 4ac$

$d = (-7)^2 - 4(1)(15)$

$= 49 - 60 = -11$

2 Imaginary Roots

55. $2x^2 - 72 = 0$

$a = 2,\ b = 0,\ c = -72$

$b^2 - 4ac = 0^2 - 4(2)(-72)$

$d = 576$ Since $\sqrt{576} = 24$

2 Rational Roots

56. $-x^2 + 9x + 23 = 0$

$a = -1,\ b = 9,\ c = 23$

$d = 9^2 - 4(-1)(23)$

$= 173$

2 Irrational Roots

Solve using the most appropriate method. Simplify all irrational and complex solutions.

57. $2x^2 - 18 = 78$ (Sq. Root method)

$x^2 - 9 = 39$

$x^2 = 48$

$x = \pm\sqrt{48}$

$x = \pm 4\sqrt{3}$

48

16 3

58. $x^2 - 16x + 73 = 0$ (Completing the Square Method)

$(x - \ )(x - \ ) = 0$

$x^2 - 16x + \underline{64} = -73 + \underline{64}$

$(x - 8)^2 = -9$

$x - 8 = \pm 3i$

$\boxed{x = 8 \pm 3i}$

73

-16

59. $9x^2 - 18x = 11$ (Complete Square)

$9x^2 - 18x - 11 = 0$

$9x^2 - 18x + \_\_ = 11 + \_\_$

$9(x^2 - 2x + \underline{1}) = 11 + \underline{9}$

$9(x - 1)^2 = 20$

$(x - 1)^2 = \frac{20}{9}$

$x - 1 = \pm\sqrt{\frac{20}{9}}$

$x = 1 \pm \frac{\sqrt{20}}{3} \rightarrow \boxed{1 \pm \frac{2\sqrt{5}}{3}}$

60. $2x^2 - 10x - 26 = x - 5$ (Factor)

$2x^2 - 11x - 21 = 0$

$\boxed{(2x + 3)(x - 7)}$

$3x$

$\pm 14x$

$\boxed{-11x}$ ✓

61. The roots of a quadratic equation are -7 and 1. Write an equation that could represent this function. Give your answer in standard form and factored form.

$x = -7,\ x = 1$

$= \boxed{(x + 7)(x - 1) = 0}$ (F.F.)

or $(x + 7)(x - 1) = 0$

$= x^2 - x + 7x - 7 \rightarrow x^2 + 6x - 7 = 0$ (S.F.)

Topic 13: Applications & Regression

79. This past Saturday, Jack worked for 3 hours mowing the lawn and doing chores in the house. Mowing the lawn burns 352 calories per hour and doing chores burns 136 calories per hour. If he burned a total of 570 calories, how long did it take him to mow the lawn?

Let X = Time (Hr) Mowing
Y = Time (Hr) Chores

$X + y = 3 \rightarrow y = 3 - x$

$352x + 136y = 570 \rightarrow 352x + 136(3 - x) = 570$

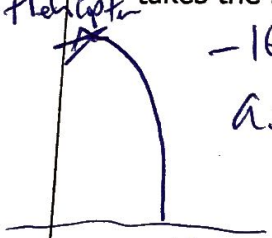
$352x + 408 - 136x = 570$

$216x = 162 \rightarrow x = \frac{162}{216} = 0.75$

Takes Jack 0.75 Hr or 45 Min To mow the lawn

80. The pilot in a helicopter tossed a life float down to a swimmer in the water. The height, h, of the float after time t seconds is represented by the equation $h(t) = -16t^2 - 3t + 55$. Find the time it takes the float to reach the swimmer.

Helicopter



$-16t^2 - 3t + 55 = 0$

$a = -16, b = -3, c = 55$

$t = \frac{3 \pm \sqrt{9 - 4(-16)(55)}}{2(-16)}$

$= \frac{3 \pm \sqrt{3520}}{-32} = \frac{3 \pm 59.33}{-32}$

$\cancel{t = -1.95}$ Not In Domain

$t = 1.76$ Sec

Takes 1.76 Sec for The Float to

H20 when $H(t) = 0$

Topic 14: Functions

Use $f(x) = x^2 + 5x - 14$, $g(x) = x + 1$, and $h(x) = 15 - 2x^2$ to answer questions 83 – 88.

83. Find $[f(7)] - 2$

$f(7) = 7^2 + 5(7) - 14$

$= 49 + 35 - 14$

$= 84 - 14 = 70$

$[f(7)] - 2 = 70 - 2 = 68$

84. If $g(x) = -23$, find x.

$g(x) = x + 1$

$-23 = x + 1 \rightarrow x = -23 - 1$

$\boxed{x = -24}$

85. Find $(f - g)(x) = f(x) - g(x)$

$(f - g)x = (x^2 + 5x - 14) - (x + 1)$

$= x^2 + 5x - 14 - x - 1$

$= \boxed{x^2 + 4x - 15}$

86. Find $(f \cdot h)(x) = f(x) \cdot h(x)$

$= (x^2 + 5x - 14)(15 - 2x^2)$

$= -2x^4 - 10x^3 + 43x^2 + 75x - 210$

	x^2	$5x$	-14
$-2x^2$	$-2x^4$	$-10x^3$	$28x^2$
15	$15x^2$	$75x$	-210

85. Find $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{x + 1}{x^2 + 5x - 14}$

or $\frac{x + 1}{(x + 7)(x - 2)}$

88. Find $(h \circ g)(x)$

$= h(g(x)) = h(x + 1)$

$= 15 - 2(x + 1)^2$

$= 15 - 2(x^2 + 2x + 1) = 15 - 2x^2 - 4x - 2$

$= -2x^2 - 4x + 13$

Use $f(x) = x^2 + 5x - 14$, $g(x) = x + 1$, and $h(x) = 15 - 2x^2$ to answer questions 89 – 90.

89. Find $(f + h)(-2)$ $= f(-2) + h(-2)$

$f(-2) = (-2)^2 + 5(-2) - 14$

$= 4 - 10 - 14 = -20$

$h(-2) = 15 - 2(-2)^2 = 15 - 8 = 7$

$(f+h)(-2) = -20 + 7 = \boxed{-13}$

90. Find $g(f(8))$ $\Rightarrow$

$f(8) = 8^2 + 5(8) - 14$

$= 64 + 40 - 14 = 90$

$g(f(8)) = g(90) = 90 + 1 = 91$

Topic 7: Absolute Value Equations & Inequalities

Solve each equation. Check all solutions.

50. $|4m + 5| = 9$

$4m + 5 = 9$

$4m = 4$

$m = 1$ ✓

or

$4m + 5 = -9$

$4m = -14$

$m = \frac{-14}{4}$

$m = -7/2$ ✓

Check:

$|4(1) + 5| \stackrel{?}{=} 9$

$|4 + 5| \stackrel{?}{=} 9$

$9 = 9$ ✓

check:

$|4(\frac{-7}{2}) + 5| \stackrel{?}{=} 9$

$|-14 + 5| \stackrel{?}{=} 9$

$|-9| \stackrel{?}{=} 9$

$9 = 9$ ✓

51. $-5|7 - x| + 6 = -14$

$-5|7 - x| = -20$

$|7 - x| = 4$

$7 - x = 4$ OR $7 - x = -4$

$-x = -3$

$x = 3$ ✓

$-x = -11$

$x = 11$ ✓

check

$-5|7 - 3| + 6 \stackrel{?}{=} -14$

$-5|4| + 6 \stackrel{?}{=} -14$

$-14 = -14$ ✓

Check

$-5|7 - 11| + 6 \stackrel{?}{=} -14$

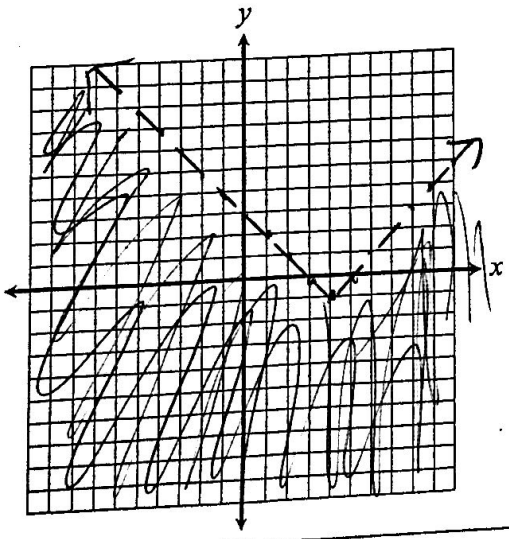
$-5|-4| + 6 \stackrel{?}{=} -14$

$-20 + 6 = -14$ ✓

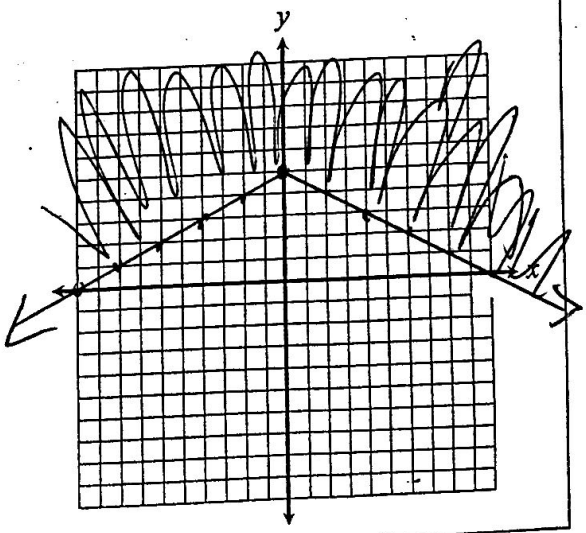
Topic 12: Graphing Absolute Value Inequalities

Graph each inequality.

77. $f(x) < |x - 4| - 1$ Vertex $(4, -1)$

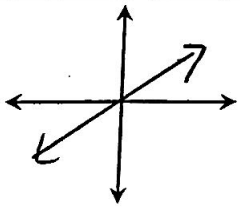
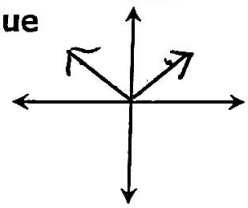
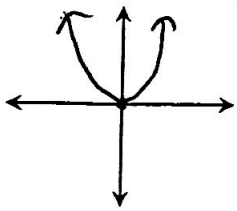
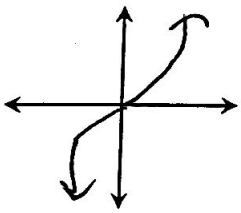
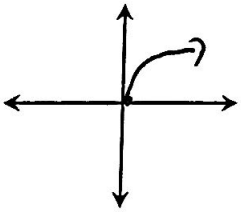
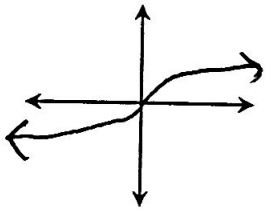


78. $f(x) \geq -\frac{1}{2}|x| + 5$ Vertex $(0, 5)$



Topic 11: Parent Functions, Transformations, and Graphing

Identify each parent function, then sketch the graph.

Linear	Absolute Value	Quadratic
$y = x$ 	$y = \|x\|$ 	$y = x^2$ 
Cubic	**Square Root**	**Cube Root**
$y = x^3$ 	$y = \sqrt{x}$ 	$y = \sqrt[3]{x}$ 

For each equation: (1) identify the function family, and (2) describe the transformations.

70. $f(x) = -2\sqrt{x-9}$

Square root family

Shift Right 9; Reflect over X-axis; Vertical stretch by a factor of 2.

71. $f(x) = \frac{1}{3}(x+7)^2 - 2$ Quadratic Family

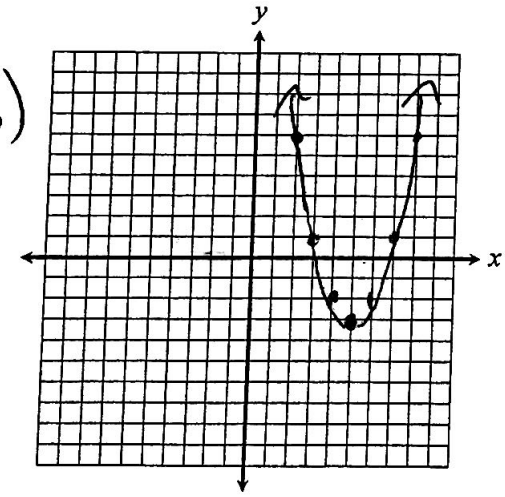
Shift Left 7, down 2

Vertical compression by a factor of 1/3.

Graph each function and identify its key characteristics.

72. $f(x) = (x-5)^2 - 3$

Vertex $(5, -3)$



Domain: All Real Numbers

Range: $y \geq -3$

Vertex: $(5, -3)$

End Behavior: As $x \to \infty$, $f(x) \to$ ∞

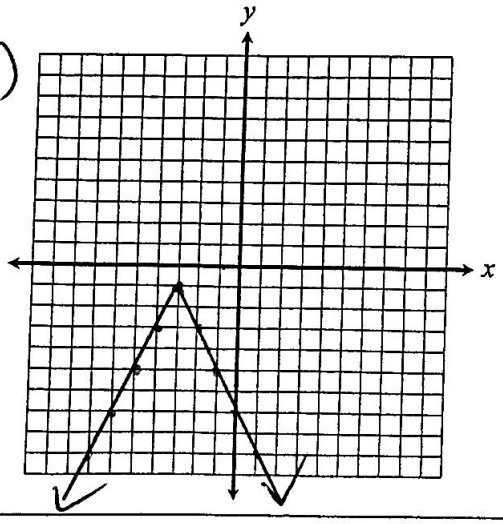
As $x \to -\infty$, $f(x) \to$ ∞

Inc. Intervals: $(5, \infty)$

Dec. Intervals: $(-\infty, 5)$

73. $f(x) = -2|x+3| - 1$

Vertex $(-3, -1)$



Domain: $\mathbb{R}$

Range: $y \leq -1$

Vertex: $(-3, -1)$

End Behavior: As $x \to \infty$, $f(x) \to$ $-\infty$

As $x \to -\infty$, $f(x) \to$ $-\infty$

Inc. Intervals: $(-\infty, -3)$

Dec. Intervals: $(-3, \infty)$