AP Calculus 3.1–3.3 Review's ANSWERS

① $m_{sec} = \dfrac{f(a+\Delta x)-f(a)}{\Delta x}$ | a) $m_{sec} = \dfrac{f(x)-f(a)}{x-a}$ | a) $m_{sec} = \dfrac{f(a+h)-f(a)}{h}$

$m_{Tan} = \lim\limits_{\Delta x \to 0} \dfrac{f(a+\Delta x)-f(a)}{\Delta x}$ | b) $m_{Tan} = \lim\limits_{x \to a} \dfrac{f(x)-f(a)}{x-a}$ | b) $m_{Tan} = \lim\limits_{h \to 0} \dfrac{f(a+h)-f(a)}{h}$

② $f(x) = \begin{cases} x+3, & x+3 \geq 0 \\ -(x+3), & x+3 < 0 \end{cases} \rightarrow \begin{cases} x+3, & x \geq -3 \\ -x-3, & x < -3 \end{cases}$

③ No, $f(x)$ is not differentiable at $x=-3$ because

$f'(x) = \begin{cases} 1, & x \geq -3 \\ -1, & x < -3 \end{cases}$ The derivatives of each side of -3 are different.

④ $f(x) = x^2 + x$

$m = f'(x) = \lim\limits_{h \to 0} \dfrac{(x+h)^2 + (x+h) - (x^2+x)}{h} = \lim\limits_{h \to 0} \dfrac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h \cancel{- x^2} \cancel{- x}}{h}$

$f'(x) = \lim\limits_{h \to 0} \dfrac{\cancel{h}(2x+h+1)}{\cancel{h}} = \boxed{2x+1}$

⑤ $y - 2 = 3(x-1)$ or $y = 3x - 1$

⑥ will do in class ⑦ D ⑧ E

⑨ a: cusp ; d, h: Discontinuous ; g: Vertical Tangent

⑩ $y - 5 = \frac{7}{2}(x-2)$ or $y = \frac{7}{2}x - 2$

(11) a) -12 ; b) 0 ; c) $-\frac{3}{4}$ d) 48

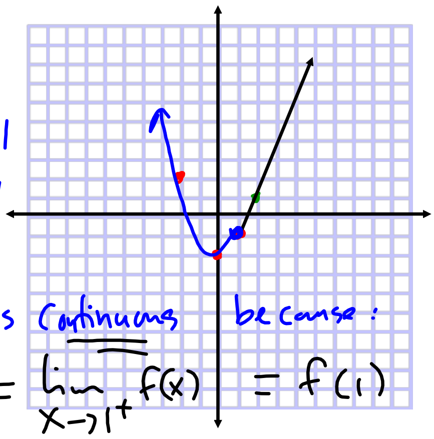
(12) $(2,-3)$ (13) 0 (14) $f'(3) = 10$

or $f'(x)\Big|_{x=3} = 10$

(15) $\dfrac{d^2y}{dx^2} = \dfrac{2}{(x-2)^3}$

(16) $B \to 1$
$D \to 2$
$F \to 3$
$E \to 4$
$C \to 5$
$A \to 6$

#6



$f(x) = \begin{cases} x^2 - 2, & x \le 1 \\ 2x-3, & x > 1 \end{cases}$

(6b) f(x) is continuous because:

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$1^2 - 2 = 2(1) - 3 = 1^2 - 2$

$-1 = -1 = -1$ ✓

f(x) is diff. because:

$f'(x) = \begin{cases} 2x, & x \le 1 \\ 2, & x > 1 \end{cases} \Rightarrow \begin{array}{l} x \to 1^-: f'(x) = 2 \\ x \to 1^+: f'(x) = 2 \end{array}$

Since $f'(x) = 2$ as $x \to 1^-$ and $x \to 1^+$.

Therefore $f(x)$ is diff at $x = 1$.

15)

$$y=\frac{1}{x-2}$$

$$\frac{d^{2}y}{dx^{2}}=?$$

Same as $y''=?$

$$\frac{Lo\,D_i\,Hi-Hi\,D_i\,Lo}{Lo^2}$$

$$y'=\frac{0-(1)(1)}{(x-2)^2}=\frac{-1}{(x-2)^2}$$

$$y'=\frac{-1 \leftarrow Hi}{x^2-4x+4}$$

$$y''=\frac{0-(-1)(2x-4)}{((x-2)^2)^2}=\frac{2\cancel{(x-2)}}{(x-2)^{\cancel{4}\,3}}=\boxed{\frac{2}{(x-2)^3}}$$

11) $f(2)=6$ $f'(2)=-3$ $g(2)=2$ $g'(2)=-1$

A: find $h'(2)$ if $h(x)=f(x)g(x)$

$$h'(x)=(f'(x))(g(x))+(f(x))(g'(x))$$

$$h'(2)=(f'(2))(g(2))+(f(2))(g'(2))$$

$$h'(2)=(-3)(2)+(6)(-1)$$

-12

11) $f(2)=6 \quad f'(2)=-3 \quad g(2)=2 \quad g'(2)=-1$

D) Find $g'(2)$ if $g(x)=x^3f(x)$

$$g'(x)=x^3f'(x)+f(x)3x^2$$

$$g'(2)=2^3f'(2)+f(2)3(2)^2$$

$$g'(2)=8(-3)+6(12)$$

$$g'(2)=-24+72=\boxed{48}$$

10) $\underbrace{\text{eq of tangent line}}$ to the curve $f(x)=\frac{x^3+2}{x}$ $(2, f(2))$

Slope $= m = y' = f'$

$$f'(x)=\frac{x(3x^2)-(x^3+2)(1)}{x^2}$$

Pt$(2, 5)$

$$f(2)=\frac{2^3+2}{2}=\frac{10}{2}=5$$

$$f'(2)=\frac{2(3(2)^2)-(2^3+2)}{2^2}=\frac{24-10}{4}=\frac{14}{4}=7/2 \leftarrow m$$

$$y-5=\frac{7}{2}(x-2)$$

11) $f(2)=6$ $f'(2)=-3$ $g(2)=2$ $g'(2)=-1$

A: find $h'(2)$ if $h(x)=f(x)g(x)$

c) Find $h'(2)$ if $h(x)=\dfrac{f(x)}{g(x)}$

$$h'(x)=\frac{(g(x))(f'(x))-(f(x))(g'(x))}{(g(x))^2}$$

$$h'(2)=\frac{(g(2))(f'(2))-(f(2))(g'(2))}{(g(2))^2}$$

$$h'(2)=\frac{(2)(-3)-(6)(-1)}{4}$$

$$h'(2)=\frac{-6++6}{4}=0$$

(14) $\lim\limits_{x\to \boxed{3}}\dfrac{f(x)-f(3)}{x-3}=10$

a

$\boxed{f'(3)=10}$

Remind:

$f'(a)=\lim\limits_{x\to a}\dfrac{f(x)-f(a)}{x-a}$

$\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=f'(x)$

what if: $\boxed{\lim\limits_{h\to 0}\dfrac{f(5+h)-f(5)}{h}=6}$?

$=f'(5)=6$

12) $f(x) = x^2 - 4x + 1$

$f'(x) = 2x - 4 = 0$

$2x = 4$

$x = 2$

$f(2) = (2)^2 - 4(2) + 1$

$f(2) = 4 - 8 + 1 = -3$

$\boxed{\text{Has hor. tangent at: } (2, -3)}$

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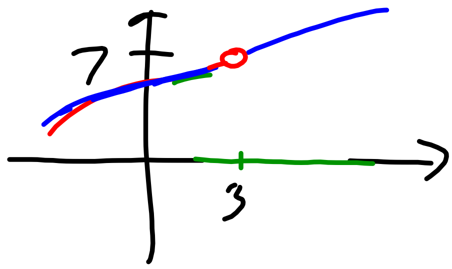
$\lim_{x \to 3} f(x) = 7$

E



HW 3.3:

#46 a, b, n, R, T are constant P: y V: x

Find $\frac{dP}{dV}$

$$P = \frac{nRT}{V-nb} - \frac{an^2}{V^2}$$

constants

$$\frac{dP}{dV} = \frac{0-(nRT)(1-0)}{(V-nb)^2} - \frac{-an^2(2V)}{(V^2)^2}$$

$$= \frac{-nRT}{(V-nb)^2} + \frac{2an^2}{V^3}$$

31) $$y = \frac{\sqrt{x}-1}{\sqrt{x}+1}$$

$y = \sqrt{x} = x^{\frac{1}{2}}$

$y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$$\frac{dy}{dx} = \frac{(\sqrt{x}+1)\left(\frac{1}{2\sqrt{x}}\right) - (\sqrt{x}-1)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}+1)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}\left[(\sqrt{x}+1) - (\sqrt{x}-1)\right]}{(\sqrt{x}+1)^2} = \frac{\cancel{\sqrt{x}}+1-\cancel{\sqrt{x}}+1}{2\sqrt{x}(\sqrt{x}+1)^2}$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{x}(\sqrt{x}+1)^2} = \boxed{\frac{1}{\sqrt{x}(\sqrt{x}+1)^2}}$$

$x=0 \quad u(0)=\underline{5} \quad u'(0)=\underline{-3} \quad v(0)=-1 \quad v'(0)=\underline{2}$

23) a) $\frac{d}{dx}(uv)$

at $x=0$

$= u \cdot v' + v \cdot \underline{u}'$

$= 5 \cdot (2) + (-1)(-3) = 10 + 3 = \boxed{13}$

48

$R = M^2\left(\frac{C}{2} - \frac{M}{3}\right) = \frac{CM^2}{2} - \frac{M^3}{3}$

Treat M as x:

Find $\frac{dR}{dM}$

Constant: C, M

$\frac{dR}{dM} = \frac{\cancel{2}CM}{\cancel{2}} - \frac{\cancel{3}M^2}{\cancel{3}} = \boxed{CM - M^2}$